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Numerical methods

MME308

Assignment 5

Grop.

Problem no: 1,8,11,15

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PROBLEM 1

GIVES

    a) Y= x3 + 4x -15   at x=0 h= 0.1
    b) Y=x2cosx   at x=0.5 h=0.5

Required

Compute the fires f'(x) and second f''(x) order central difference approximation of O(h2)
Solution

    a) Y= x3 + 4x -15   at x=0 h= 0.1

X	-0.2	-0.1	0	0.1	0.2
y	-15.808	-15.401	-15	-14.599	-14.192

The first central difference approximation is given by :

$$F'(x) = \frac{f(x+h)-f(x-h)}{2h} + O(h^2)$$

$$F'(0) = \frac{f(0.1)-f(-0.1)}{2h}$$

$$F'(0) = \frac{-14.599+15.401}{2(0.1)}$$

$$F'(0) = 4.01$$

The second difference approximation is given by:

$$F''(x) = \frac{f(x+h)-2f(X)+f(x-h)}{h2}$$

$$F''(x) = \frac{f(0.1)-2f(0)+f(-0.1)}{h2}$$

$$F''(x) = \frac{-14.599+2*15-15.401}{h2}$$

$$F''(0) = 0$$

    b) Y =x^{2}cosx   at x =0.5 h=0.5

X	-0.5	0	0.5	1	1.5
y	0.24999	0	0.24999	0.99985	2.24923

The first central difference approximation is given by :

$$F'(x) = \frac{f(x+h)-f(x-h)}{2h} + O(h^2)$$

$$F'(0.5) = \frac{f(1)-f(0)}{2h}$$

$$F'(0.5) = \frac{0.99985 - 0.}{2(0.5)}$$

$$F'(0.5) = 0.99985$$

The second difference approximation is given by:

$$F''(x) = \frac{f(x+h)-2f(X)+f(x-h)}{h2}$$

$$F''(0.5) = \frac{f(1)-2f(0.5)+f(0)}{h2}$$

$$F''(0.5) = \frac{0.9999+2*0.24999-0}{h2}$$

$$F''(0) = 5.9995$$

Problem 8

Gives

X	0	0.5	0.75	1	1.15	1.25	1.5	2.25
y	0	0.375	0.844	1.5	1.984	2.344	3.375	7.594

Required :

T=0 , 1.25 , 2.25 by use numerical differential.

Solution :

Forward $\quad$ **F'(x) =** $\dfrac{-3f(x)+4f(x+h)-f(x+2h)}{2h}$ $+$ O(h^2)

forward $\quad$ **F'(0) =** $\dfrac{-3f(0)+4f(0.5)-f(1)}{2h}$ $+$ O(h^2)

Forward $\quad$ **F'(0) =** $\dfrac{-3*0+4*0.375-1.5}{2(0.5)}$ $+$ O(h^2)

F'(0) $_{= 0}$

center $\quad$ **F'(x) =** $\dfrac{f(x+h)-f(x-h)}{2h}$ $+$ O(h^2)

center $\quad$ **F'(1.25) =** $\dfrac{f(1.5)-f(1)}{2h}$ $+$ O(h^2)

center $\quad$ **F'(1.25) =** $\dfrac{5.625-2.5}{2(0.25)}$ $+$ O(h^2)

F'(0) = 6.25

Backward $\quad$ **F'(x) =** $\dfrac{3f(x)-4f(x-h)+f(x-2h)}{2h}$ $+$ O(h^2)

Backward $\quad$ **F'(2.25) =** $\dfrac{3f(2.25)-4f(1.5)+f(0.75)}{2h}$ $+$ O(h^2)

Backward $\quad$ **F'(2.25) =** $\dfrac{3*7.594-4*3.375+0.844}{2(0.75)}$ $+$ O(h^2)
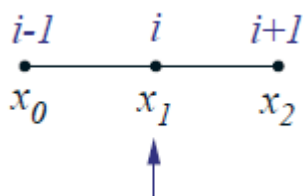
F'(0) = 6.7507

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<u>problem 11</u>

Given :

Taylor series :

$$F(x)= f(x_0) + f'(x)(x-x_0) + \frac{f''(x0)}{2!}(x-x0)^2 + \ldots\ldots + f^{(n)}(X-X0)^n$$

$X_i - X_{i-1} = 3h$ and $X_{i+1} - X_i = h$

<u>Required :</u>
Find f'(x) , A and B

Form the Taylor series :

$$F(x + h) = f(x) + h\, f'(x) + \frac{h2}{2!} f''(x) + F^n(x) + \ldots\ldots \qquad (1)$$

$$F(x-3h) = f(x) - 3h f'(x) + \frac{(3h)2}{2!} f''(x) + \ldots\ldots \qquad (2)$$

بطرب المعادلة (1) في 3^2 تصبح :

$$9\, F(x + h) = 9\, f(x) + 9\, h\, f'(x) + 9\, \frac{h2}{2!} f''(x) + \ldots\ldots \qquad (3)$$

بطرح المعادلة (2) من (3)

$$9f(x + h) - f(x - 3h) = 8\, f(x) + 12h\; f'(x) + O(h^2)$$

$$12h\, F'(X) = 9\, f(x+ h) - f(x - 3h) - 8\, f(x)$$

$$F'(x) = \frac{9(x+h) - f(x-3h) - 8f(x)}{12\,h} \qquad (4)$$

$$F'(x) = A\, f(x - 3h) + B\, f(x) + C\, f(x + h) \qquad (5)$$

مقارنة المعادلة 4 مع المعادلة 5 نجد ان:

$$A = -\frac{1}{12h}$$

$$B = -\frac{8}{12\,h} = -\frac{2}{3\,h}$$

$$C = \frac{9}{12\,h} = \frac{3}{4\,h}$$

Problem 15

	t	y	dy\dt	LOG(dy\dt)	Log(y)	x2	x*y
1	5	2.45	-0.071	-2.6451	0.8961	0.8030	-2.3702
2	15	1.74	-0.061	-2.7969	0.5539	0.3068	-1.5492
3	25	1.23	-0.043	-3.1466	0.2070	0.0429	-0.6514
4	35	0.88	-0.0305	-3.4900	-0.1278	0.0163	0.4461
5	45	0.62	-0.022	-3.8167	-0.4780	0.2285	1.8245
6	55	0.44	-0.018	-4.0174	-0.8210	0.6740	3.2982
Σ				-19.9126	0.2301	2.0715	0.9981

Solution

$$- \frac{dy}{dt} = k\, y^{\,n}$$

$$\text{Log}(- \frac{dy}{dt}) = n \log(y) + \log(k)$$

$$Y = A\,X + B$$

$$\sum X = 0.2301 \quad \sum Y = -19.9126 \quad \sum X^2 = 2.0715 \quad \sum X.Y = 0.9981 \quad N = 6$$

$$aN \;+\; b \sum_{i=1}^{n} xi = \sum_{i=1}^{n} yi \left. \vphantom{\begin{array}{c}1\\2\\3\end{array}} \right\}$$

(1)

$$a\sum_{i=1}^{n} xi + b\sum_{i=1}^{n} xi2 = \sum_{i=1}^{n} xiyi$$

Now, eq (1) becomes .

$$6A \;+\; 0.2301\, B = -\,19.9126 \left. \vphantom{\begin{array}{c}1\\2\\3\end{array}} \right\}$$

(1)`

$$0.2301\, A \;+\; 2.0715\, B = 0.9981$$

in this system is solving by using guess elimination

$$\begin{pmatrix} 6 & 0.2301 \\ 0.2301 & 2.0715 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -19.9126 \\ 0.9981 \end{pmatrix}$$

R1= R1/6

$$\begin{pmatrix} 1 & 0.0384 \\ 0.2301 & 2.0715 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -3.3188 \\ 0.9981 \end{pmatrix}$$

R2= R2-0.2301*R1

$$\begin{pmatrix} 1 & 0.0384 \\ 0 & 2.0627 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -3.3188 \\ 1.7618 \end{pmatrix}$$

R2= R2/2.0627

$$\begin{pmatrix} 1 & 0.0384 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -3.3188 \\ 0.8541 \end{pmatrix}$$

Now, this augmented matrix represents the equivalent linear system.

1 A + 0.0384 B = -3.3188   (2)

B =   0.8541   (3)

Since B = 0.8541 from the last equation ,substituting in the equation (2) by B

A + 0.0384 B =-3.3188

A + 0.0384(0.8541)=-3.3188

That is,   *A=-3.3516*

but A = ln (K) ,K= e^A ,   B = n

Hence , the solution set consists of **k=0.0350   , n=0.8541**

Hence the fitting

$$-\frac{dy}{dx} = K\ y^{n}$$

$$Y = 0.0350\ X^{0.8541}$$